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ABSTRACT

This paper presents evidence to indicate that spontaneously generated analogies can play a significant role in the problem solving process of scientifically trained individuals. In addition, it is suggested that these individuals exhibit more than one method for generating analogies. Ten scientists (representing physics, mathematics, and computer science), were asked to solve a problem involving a spring and weight. Subjects were asked to verbalize their thought processes, which were audiotaped. An analysis of the results indicates that analogies can be generated by an associative process which triggers the new involvement of an old but separate idea, or they can be generated by a transformation process which modifies the current problem situation. Both generation processes are considered to be creative acts, and may be important sources of creative power in scientific thinking. In contrast to the common view of analogous cases, this study indicates that the novelty of a number of the analogies generated by a transformation suggests that they are newly constructed cases, rather than cases recalled directly from memory. These findings may have significant educational implications for the learning of scientific models and the transfer of knowledge to new situations. Numerous figures are supplied. (TW)

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OBSERVED METHODS FOR GENERATING ANALOGIES IN SCIENTIFIC PROBLEM SOLVING

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ABSTRACT

Evidence is presented indicating that spontaneously generated analogies can play a significant role in the problem solving of scientifically trained subjects. Furthermore, subjects exhibit more than one method for generating analogies. Apparently analogies can be generated by an associative process which triggers the new involvement of an old but separate idea, or they can be generated by a transformation process which modifies the current problem situation. Both generation processes are creative acts, and may be important sources of creative power in scientific thinking. A common view of an analogous case is that it is a conception sitting in long term memory which is accessed at some point and compared to the problem situation. However, the novelty of a number of the analogies generated via a transformation suggests that they are newly constructed cases rather than cases recalled directly from memory. These findings have significant educational implications for the learning of scientific models and the transfer of knowledge to new situations.

OBSERVED METHODS FOR GENERATING ANALOGIES IN SCIENTIFIC PROBLEM SOLVING

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A number of researchers have discussed the important role of analogical reasoning in science [6, 7, 8, 10] and education [11, 12, 13, 14]. In science, it has been argued that analogies can play an important role in the creation of new theoretical hypotheses. In some cases these hypotheses can become established analogue models, such as the "billiard ball" model for gases. In education, analogical reasoning may be important in the learning of such models and in the transfer of learned knowledge to new, unfamiliar situations.

Analogical reasoning has traditionally been considered something of an enigma, and as a cognitive process it seems particularly resistant to a full explication. It represents a style of reasoning which seems to fall well outside the domain of deductive logic. There is a certain gamble involved in using an analogy; initially there is no guarantee whatsoever that the analogy will be at all helpful. Yet, analogies can sometimes lead to strikingly elegant solutions and to powerful explanations.

Previous investigations have related analogical reasoning to problem solving [15, 16], measures of intelligence [17], and the development of concepts [18]. This paper describes the spontaneous use of analogies in problem solving by scientifically trained subjects. The data come from video tapes of subjects solving problems aloud. The major questions addressed are: (1) Can one document the spontaneous use of analogies in the problem solving of expert scientists? (2) Are spontaneous analogies generated in more than one way? Where do they come from? (3) Are analogous cases always "recalled" or are they sometimes invented?

The problem that was used in this study is shown in Figure 1. A spontaneous analogy is said to occur when the subject spontaneously shifts his attention to a situation B which differs in a significant way from the original problem situation A and seeks to apply findings from B to A. (A more precise definition will be developed below.) An example of an analogy for the spring coils problem would be to think about the weights being hung vertically from long and short straight elastic bands instead of wide and narrow springs. Knowing that the larger band will stretch more might suggest that the wider spring will stretch more. As a second example, one subject thought about the saw blade shown in Figure 2. He felt that a long blade would bend more easily than a short one, and this indicated to him that the wider spring might stretch more. In what follows it will sometimes be convenient to distinguish between two parts of an analogy, the analogous *case* and the analogy *relation*. The analogous case in the above example is the saw blade experiment, and the analogy relation is the relationship being proposed by the subject of a partial equivalence between the original case involving springs and the analogous case involving saw blades

Overview. The major points to be made in this paper are as follows. (1) Evidence is presented indicating that spontaneously generated analogies can play a significant role in the problem solving of scientifically trained subjects [See also references 1, 2, 3]. (2) Several hypotheses about mechanisms for generating analogies are considered. An initial hypothesis is that analogies are generated by first thinking of a formal abstract principle at a higher level which applies to the problem (such as an equation or one of Newton's Laws) and then generating another example of the principle. Almost no evidence was found to support this hypothesis. (3) Instead, a pattern was often observed of generating analogies directly via an associative leap from the problem situation A to a second situation B at the same concrete level. In this case, a subject is "reminded" of a second situation B which shares one or more features with A (4) However, associative leaps were not the only analogy generation method observed. Another pattern was observed where a subject generates an analogy via a transformation which "warps" or modifies the original situation A to produce the analogous situation B. The majority of the analogies generated appeared to fit this pattern. (5) Hypotheses concerning the cognitive mechanisms responsible for the observed analogy generation patterns are proposed. (6) A common view of an analogous case is that it is a conception sitting in long term memory which is accessed at some point and compared to the problem situation

However, the novelty of a number of the analogies generated via transformations suggested that they were newly constructed cases rather than cases recalled directly from memory.

Steps in using an analogy. In a previous report I presented two case studies which contained evidence countering the idea that solutions by analogy are "instant solutions" that are achieved very quickly with minimal effort. [2, 3] In fact these studies suggest that there are several important steps in making an inference by analogy, and these steps (particularly steps (b) and (c) below) can require a significant length of time and significant effort to accomplish. The steps are:

(a) *Generating the analogy*. The analogous conception B is generated, or "comes to mind".

(b) *Confirming the analogy relation*. The validity of the analogy relation is examined critically and must be confirmed.

(c) *Comprehending the analogous case*. The subject examines his or her understanding of the analogous case critically, and the analogous case becomes well-understood, or at least predictive.

(d) *Transferring findings*. The subject transfers conclusions or methods from B back to A.

This study focuses on step (a) above, the process of analogy generation. An analysis of steps (b), (c) and (d), as well as detailed case studies of two subject's protocols from the spring problem are given in [2] and summarized in [1] and [3].

METHOD

Ten experienced problem solvers were asked by the author to think aloud as much as possible while solving the spring problem

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above All were advanced doctoral students or professors in technical fields.[c] Subjects were recruited who had a reputation within their department for having done relatively creative work in the past. Five of the subjects were physicists, three were mathematicians, and two were computer scientists. The subjects were told that the purpose of the interview was to study problem solving methods. They were given instructions to solve the problem "in any way that you can", and were asked to give a rough estimate of confidence in their answer. Probing by the interviewer was kept to a minimum, usually consisting of a reminder to keep talking. Occasionally the interviewer would ask for clarification of an ambiguous statement. Most sessions were videotaped and all were audio taped.

The correct answer to the problem is that the wide spring will stretch farther (the stretch in fact increases with the cube of the diameter). This seems to correspond to most people's initial intuition about the problem. However, explaining *why* the wide spring stretches more (and explaining exactly where the stretch of the spring comes from) is a much more difficult task when taken seriously.

Observational definition of "spontaneous analogy". In defining criteria for recognizing a "spontaneous analogy", we want the definition (1) to include cases arising from the act of focussing on a problem situation; (2) to include such cases whether or not they ultimately contributed an answer to the problem that is different from the original problem situation; (3) to rule out trivial cases that do not involve a structural or functional similarity; and (4) where appropriate, to separate analogy generation from other problem solving processes such as generating extreme cases, breaking a solution into independent parts, and analyzing the problem in terms of a theoretical principle.

The following observational definition was used to code for the generation of a spontaneous analogy: (1) the subject, without provocation, considers another situation B where one or more features ordinarily assumed fixed in the original problem situation A are different, i.e., the analogous case B violates a "fixed feature" of A (to be defined below); (2) the subject indicates that certain structural or functional relationships (as opposed to surface features alone) may be equivalent in A and B; and (3) the related case B is described at approximately the same level of abstraction as A. For example, several subjects attempted to relate the spring problem to the analogy of comparing long and short horizontal wires or rods bent by the same weight as shown in Fig. 3. (The saw blade in Figure 2 is one variation of this analogy.) Most had a strong intuition that a long rod would bend more than a short rod. They reasoned that since the longer rod would bend more, the wider spring would probably stretch more. This analogy in fact leads to the correct prediction, and provides a plausible initial justification for it. In some cases, a more complicated analogy was constructed (such as a spring with hexagonal or square coils) which led to a more complete justification of the answer.

The above definition distinguishes between <u>fixed features</u> and <u>problem variables</u>. We assume that in a typical problem situation some features are assumed by the problem solver to be fixed--not subject to change--and other features are assumed to be changeable or manipulable. We will call the former a <u>fixed feature</u> and the latter a <u>problem variable</u>. Examples of fixed features in the spring problem are the thickness of the wire and the helical shape of the spring. Examples of problem variables are the variables of coil diameter and amount of stretch We assume that considering the problem of a horizontal rod represents a change in what was originally a fixed feature (the shape of the spring) in the subject's initial comprehension of the problem. Thus we call the bending rod an analogous case Effectively, the subject's assumptions about which aspects of the situation are fixed and which are mobile determine a stable context or problem representation within which he or she works on the problem. An analogy, then, changes the problem representation being considered.

The above definition excludes a number of types of related cases that were <u>not</u> counted as analogous cases. For example, when subjects used a simplifying partition such as looking at a spring with a single coil, it was not counted as an analogy if it consisted simply of thinking about a <u>part</u> of the original system. Secondly, the indication of a mere surface similarity, such as one subject's comment that the drawing of springs in the original problem "reminded him of eels," was not counted as an analogy Thirdly, extreme cases such as considering a very narrow or very wide spring were observed as well, but these were also not counted as analogies, because width is considered to be a problem variable. Fourth, the use of the term "analogy" was also confined to a related case B at approximately the same level of abstraction as A. This criterion rules out saying that a bird is analogous to a robin, or that a spring is analogous to the general notion of a harmonic oscillator. Thus, when one subject thought about the behavior of a door spring as a particular example of a helical spring, this was also not counted as an analogy. Further comments on the definition of analogy developed here appear in the Discussion section at the end of the paper.

RESULTS: OBSERVED ANALOGIES

The solutions collected were up to 90 minutes long. All subjects favored the (correct) answer that the wide spring would stretch farther. But the subjects varied considerably in the types of explanations they gave for their prediction. The subjects generated a large variety of analogous cases. The following transcript of S1 documents the generation of the "hairpin" analogy shown in Figure 4. (Numbers on left are transcript line numbers.)

031 S: ...The equivalent problem that might have the same answer is-- suppose I gave you the problem in a way instead of being a coiled spring, it's a long U spring like that, just like a hairpin. And now I hang a weight on the hairpin, and see how far it bends down. Now I make the hairpin twice as long with the same wire and see how far it bends down. Now that goes with the cube. That's the deflection in the length of the cantilever beam. Heh, heh--- and maybe it comes out that way with the spring. So my-- I would bet about, about 2 to 1 I would bet that the answer to this [original spring problem] is that it goes down 8 times as far.

Instances of spontaneous analogies were coded from the transcripts and video tapes using the definition given above.

The results are shown in Table 1. The subjects generated forty analogies altogether. An analogy was classified as salient if it appeared to be part of a serious attempt to generate or confirm a solution. An analogy was classified as non-salient if it was mentioned as an aside or commentary and did not appear to be an attempt to increase understanding of the problem situation. (The saliency of an analogy does not depend on whether the solution generated is correct.) Thirty-two of the analogies were salient according to this criterion, and a number of these are illustrated in Figures 2 through 18. The thirty-two salient analogies include three analogies generated by one subject, a Nobel laureate in physics, who solved the qualitative problem almost immediately, but spontaneously went on to generate analogies while determining the exponent in the relationship between diameter and stretch. Eight of the ten subjects generated at least one analogy, and seven of the ten generated at least one salient analogy. The most common species of analogy was the bending rod and variations thereof, such as a bending saw blade, a bending wire, and a diving board (Figure 5). A total of six of the subjects generated an analogy of this type. In summary, spontaneously generated analogies were observed to play a significant role in the problem solutions of these scientifically trained subjects. There was a large variety of analogies generated, and most of the analogies were salient.

OBSERVED ANALOGY GENERATION METHODS

Analysis of the transcripts indicated that there were not one, but at least three types of analogy generation methods: generation via an equation or abstract principle, generation via a "generative transformation," and generation via an "associative leap." Examples of each type are discussed below, followed by observational definitions and the observed frequency of each method.

<u>Generation from a formal principle</u>. A plausible mechanism for generating analogies can be derived from the common situation in science where a single equation or abstract principle applies to two or more different contexts, such as F = kx for a spring and for a pendulum with small displacements. This suggests that analogies may be formed by first recognizing that the original problem situation, A, is an example of an established abstract equation or principle, P. The analogous situation, B, is then recalled or generated as a second example of principle P. The basic observable characteristic indicating this generation method is the verbal report of an equation or formal abstract principle near the first reference to the analogous case B.

The hypothesis that the generation from a formal principle mechanism is the only mechanism responsible for producing analogies was proposed as a "null hypothesis" in this study, and evidence has been collected which challenges it. This hypothesis would imply that all analogies involve a formal principle that assimilates both of the analogous conceptions. It downplays analogy formation as a special process in its own right, and instead explains away the phenomenon in terms of the more classical processes of either classification in terms of an abstract category or assimilation to an abstract schema. Evidence for the generation via a formal principle pattern was observed in only one case. Instead, two other types of analogy formation processes appear to predominate, which I have called generative transformations and associative leaps.

Generative transformations. These occur when a subject creates a different problem situation B by modifying a feature of the original situation A that was previously assumed to be fixed. In these cases there is no mention of a formal principle or equation. For example, one subject, S2, "unrolls" the spring into a wire:

041 S: ...I'm going to unroll these things and see if that helps my intuition any.

042 S: ...Um, if I essentially, uh, uncoil or project the spring into a wire, it's going to look like, if this is where the endpoint is, the wire will actually go from here to here. That's if I actually unroll the wire.

Another subject generated an analogy via a transformation below while thinking about moving the weight out along a bending rod, and then about moving it down along the spring wire:

037 S: ...(Looking at a picture he has drawn of a bending rod) this rod here, as the weight moves along, it bends more... Hmmm, what if I imagined moving the weight along the spring, as I'm moving it along this [rod]...would that tell me anything? I don't know. What if the spring were twice as long? Now, that's interesting. I-I just had this recognition of an equivalence.

039 S: Now what if I recoiled the spring and made the spring twice as long.

041 S: ...instead of twice as wide?...uhhh..it seems to me pretty clear that the spring that's twice as long is going to stretch more... Now that's a-again, a kinesthetic intuition...but now I'm thinking...what happens...I'm again using a method of limits. I'm

> imagining that one applies a force closer and closer to the origin [top] of the spring and...as you get closer to the origin of the spring it hardly stretches at all...therefore, the further away you are along the spring, the more it stretches...So, a spring that's twice as long, I'm now quite sure, stretches more...Now if this is the same as a spring that's twice as wide, then that should stretch more... Uhhh, but _is_ it the same as a spring that's twice as wide?

Here the subject considers the new problem of doubling the length of the spring instead of the width (Figure 6). This attempt to use an analogous problem is of special interest because there is evidence of it having been generated via an imagined, continuous transformation-- that of imagining sliding the weight down along the helical spring wire. The inverse of this transformation also appears to be used to generate the extreme case of a very short spring. This extreme case appears to confirm his intuition and gives him a firm prediction of a result for the analogous case, but he is uncertain of whether the analogy relation is valid, and so he gives up on the analogy. Later, the same subject says:

057 S: ...There's something fundamentally wrong with my understanding of the whole spring business... would it help to view it compressionally rather than in terms of stretch, I wonder. That came from imagining myself manipulating it...

Here the subject talks about the idea of compressing the spring coming from "imagining myself manipulating it" [the spring]. Although this idea is not pursued by him at length, it again provides evidence for the role of a transformation involving dynamic imagery and internalized actions in generating a new idea. Furthermore, the transformations in the last two examples are distinguished by their playful and relatively non-goal-directed nature. During these moments the subject seems more to be trying out variations of the problem than progressing systematically toward a result. This may be typical of sections of highly creative solutions to scientific problems.

<u>Associative leaps</u>. In contrast to generating an analogy by making a single modification in a generative transformation, the subject using an "associative leap" jumps to an analogous situation that differs in many ways from the original problem but has a feature in common with original the situation. S2 generated evidence for several associative leaps in the middle of the protocol when he said: (line 63) "I feel as though I'm reasoning in circles and I think I'll make a deliberate effort to break out of the circle somehow...what else stretches... like rubber bands, molecules, polyesters.." apparently attempting to link the spring problem to other situations he knew something about. Each of these represents an apparent attempt to jump to a very different situation through the association of "stretchiness" rather than being a transformation which makes a single modification in the original problem situation. Thus associative leaps tend to produce analogies which are more "distant" from the original situation conceptually than those produced by transformations. In another example (see Fig. 7) subject S6, compared the wide and narrow springs to two blocks of foam rubber, one made with large air bubbles and one made with small air bubbles in the foam. He had a strong intuition that the foam with large air bubbles would be easier to compress. Another subject, S5, examined the relationships between coil width, coiling angle, and wire length by thinking about mountain

roads winding up narrow and wide mountains.

RESULTS: ANALOGY GENERATION METHODS

The 32 salient analogies in the 10 solutions to the spring problem were classified according to their method of generation using the following criteria:

Generation via a formal principle. Observable characteristics indicating analogy generation via a principle are: (1) the subject refers to an abstract formal principle (mathematical or verbal) near the first reference to the analogous case B; (2) the subject may also refer to case B as an "example" of a principle. [Note 1]

Associative leaps. Observable characteristics of an associative leap, in order of importance, are: (1) the subject mentions "being reminded of" or "remembering" the situation or refers to it as a "familiar" situation; (2) the analogous situation is different in many ways from the original situation. (3) The analogous case is a situation which should obviously be familiar to S (but may not necessarily be well understood by S).

Generative transformations. Observable characteristics, in order of importance, are: (1) the subject refers to modifying an aspect of situation A to create situation B; (2) the subject states that B is an invented situation he has not encountered before; (3) the novelty of the analogous case suggests that it has just been invented; (4) there is a plausible transformation which can change A into B.

Method unclear. An analogous case was placed in the category "method unclear" when there was not enough data in the protocol to make a confident classification of the generation method.

Generation Method Results. The results are shown in Table 2. The largest number of salient analogies (19 of 32) were generated via generative transformations. Twelve of the analogies were generated via an associative leap. Evidence was observed for generation via a principle in only one case.

In addition there were a significant number of invented analogies--analogous cases which were so novel as to be unlikely to have been drawn directly from the subjects' permanent memory (shown in Figs. 15 - 18). These cases include springs with polygonal coils (Fig. 15), two dimensional zig-zag springs (Fig. 16), and an experiment where the subject pits the narrow spring against the wide spring by attaching them to opposite sides of the weight (Fig. 17). The construction of the polygonal coils case was particularly important because it allowed one subject to discover the fact that torsion or twisting in the spring wire is an important mechanism for producing stretch. The last example of a Gedanken experiment is a torsionless spring coil made with many freely twisting ball bearing joints between increments and used to "test" (mentally) whether a spring could be made to work without torsion (it cannot, because the spring would collapse) (Fig. 18). The novelty of these Gedanken experiments indicates that analogous cases are not always simply recalled from memory--they may instead be constructed by the problem solver to help him resolve issues he is struggling with.

In summary, generative transformations and associative leaps were the primary analogy generation methods for which evidence was observed. Evidence for analogies generated via a formal principle occurred only rarely. This result does not rule out the possibility that the latter method may be used in scientific problem solving, but it does indicate that it may not be the most common method for generating analogies, and that the other two methods can play a significant role. In addition a number of novel analogous cases were generated that are not likely to have simply been retrieved from permanent memory.

DISCUSSION: I. THE PRESENCE OF ANALOGIES IN THE PROTOCOLS

Clinical vs anecdotal evidence. The findings will be discussed in the following order: (1) the presence of analogies in the solutions; (2) the different analogy generation methods observed; and (3) the presence of novel constructions in the solutions.

In the past, discussions of the role of analogical reasoning in science have been based largely on conjecture or on historical evidence [6, 7, 9, 10]. More recently, a few authors have based some of their ideas on reports from scientists attempting to recall the sequence of developments in their own ideas [8, 11, 12, 13]. But there has been almost no evidence collected which captures problem solvers in the act of generating a spontaneous analogy. The thinking aloud protocols analyzed in this study thus provide an initial body of more direct evidence which shows that such processes can actually be used in problem solving. Although thinking aloud protocols are far from being a perfect source of data, they are certainly less subject to distortion by the subject than retrospective-anecdotal reports, and they are also a richer and more fine-grained source of data. This allows one to determine which of the analogies are salient (in the sense of being an actual source of new ideas for the subject as opposed to an ornamental afterthought for purposed of communication); and it allows one to look at details such as different analogy generation mechanisms.

Why are analogies useful? An interesting feature of analogical reasoning lies in the paradox that by seeming to move away from a problem the subject can actually come closer to a solution. In order to use an analogy effectively one must be able to postpone working directly on the original problem and be willing to take an "investigatory side trip" with the faith that it may pay off in the end. This is a risky thing to do (especially while being recorded)--there is no guarantee that any of these side trips will make any contribution to the solution at all.

Part of the explanation to the "moving closer by moving away" paradox lies in the idea that humans appear to be constrained to build up new knowledge by starting from old knowledge. In the words of Ernest Nagel [9], scientists use established analogies in the form of models in science in order to "make the unfamiliar familiar". This is one of the legitimate functions of scientific models, in Nagel's view. In this view, getting closer to the answer by moving away from the problem can work because one is moving to an area one knows much more about, and one may then be able to transfer part of this knowledge back to the original problem. Such knowledge may predict a full answer, or it may be simply a suggested method of attack.

Finding a more familiar case would appear to be the motive of the subjects in generating associative leaps such as rubber bands, molecules, and car springs. However, it is not clear that all of the other analogies generated were more familiar cases in the sense that the subject had encountered them previously. The novelty of the analogous cases shown in Figs. 15 to 18 argues that they are not more familiar to the subject than the original helical spring. Their virtue is not that they are more familiar, but that they may be _simpler to understand_ that the helical spring. A simpler case may be analyzable in terms of physical principles or physical intuitions that were not recognized as relevant to the original case. Thus, in the context of problem solving, and perhaps also in the context of science, one might add the description "making the unfamiliar simpler" to the description "making the unfamiliar familiar" as a reason for the usefulness of analogies.

Analogy generation as a "horizontal" change in representation. An analogy can be said to involve a shift in the subject's representation of the problem. However, it is a shift of a special kind. Other instances of shifts in problem representation occur when the subject engages in abstract planning or in using symbolic representations such as equations. However, in the latter two cases the subject moves "vertically" to a more abstract representation whereas in moving to an analogous case, the subject moves "horizontally" to another problem situation at roughly the same level of abstraction. In one way, using an analogy is the most creative of these three strategies, since there is a sense that one is shifting one's attention to a different problem, not just to an abstract version of the same problem.

Developing useful boundaries for the concept of "spontaneous analogy". The definition of analogy given on page 6 is consistent with the idea that analogy generation is a creative and divergent process. The condition that the analogous case be one where "features ordinarily assumed fixed in the original situation are different" means that the subject must somehow break away from the original problem and shift his or her attention over to a significantly different problem. This may be difficult for some people to do, probably because of the difficulty involved in breaking "set"--breaking out of the assumptions built up in considering the original problem, and because problems assigned in textbooks are almost always amenable to more direct methods which stay within the original problem context.

To some, analogies such as a bending rod or a square coil may seem too similar to the original spring to be counted as "real" analogies. The issue here is largely semantic. The important questions are: "What is the form of the basic reasoning patterns being used?" and, "What are the most useful and fundamental distinctions to emphasize in constructing definitions for terms like 'analogy'?" Certainly much data has been collected on problem solving where no analogies occur. What seems to distinguish spontaneous analogies when they happen, more than anything else, is the fact that the subject is somehow bold enough to break away from the previous assumptions about the problem context. It is worth noting that, even though one might call the hexagonal coil idea (Fig. 15) a "close" or "non-remote" analogy, this very fruitful idea that led to a genuine scientific insight. (The hexagonal and square coil models were used by one subject to discover the major contribution to displacement of a torsion effect in the spring wire. The torsion effect can be seen in Fig. 15c by viewing rod 1 as a wrench which puts a twist in rod 2 when the end of rod 1 is pulled downward. Similarly, rod 2 twists rod 3, and so on. Twisting is in fact the predominant source of stretching, and its identification constitutes the discovery of a new causal variable in the system. The square coil can also can be used to predict that the stretch varies with the cube of the coil diameter.) Just because the analogy appears to be "close" to the original problem from hindsight does not mean that the assumption-breaking act of generating it was easy, by any means. The polygonal coil idea was generated by only one of the seven subjects generating analogies for this problem. Thus we take the act of moving to "considering a situation B which violates one or more fixed features of A" as central to the definition of a spontaneous analogy. This seems to be a more important criterion than requiring case B to have many surface features that are different from A's, and so we include cases like the hexagonal coil as examples of analogies. Such "close" analogies appear to be one of the most fruitful and powerful types of analogy observed. The definition is still fairly restrictive, however, since it excludes various extreme cases and simple partitioned parts of the problem.

II. ANALOGY GENERATION MECHANISMS

Rejection of an Initial Hypothesis. Prior to this study, the author had asked several physicists (who were not subjects) about whether they thought analogies played a role in scientific thinking. A common response was that an analogy can be used in a way which corresponds to the analogy generation via a formal principle method described above. They indicated that one sees cases in physics where two quite different fields of inquiry develop theories in the form of an equation(s), and it is then realized that the equations are of the same form. It can then be said that certain situations in the two fields are analogous in certain ways because they obey the same equation. Methods used to make progress in one field may then be attempted in the other field. To a physicist this is an important insight which has heuristic value and serves to unify and interconnect physical theories. However, this rather limited view of analogy makes its role secondary to that of mathematical analysis. Recognition of the analogy depends on the prior development of two mathematical theories. The analogy is not involved in the earlier stages of discovery but comes "after the fact"--that is, after extensive theory development has taken place.

However, the fact that in this study almost no evidence was observed for the use of formal principles in generating an analogy indicates that this is not the only role of analogies in scientific thinking. The associative leap and generative transformation methods observed suggest that analogy can play a role at a much earlier stage of the theory development process. Thus the idea that generation via a formal principle is the only method for generating analogies during problem solving played the role of a null hypothesis which was rejected in this study.

Cognitive models of analogy generation. It is interesting to propose more explicit theoretical hypotheses concerning the cognitive processes that underly each of the analogy generation methods observed. In the following paragraphs models are proposed for cognitive processes that could account for the three methods.

Generation via a formal principle. The basic observable characteristic of this method is the verbal report of an equation or abstract principle near the first reference to the analogous case B. This can be explained by assuming that an established, abstract schema P in memory is activated by some aspect of the original problem A, as shown in fig. 19a. P then in turn activates a less abstract schema B (a familiar example of P) in memory, thereby generating a proposed analogy between A and B.

Generative transformations. The strongest indicator of the generative transformation pattern occurs when the subject refers to modifying an aspect of situation A to create situation B. It is hypothesized that a generative transformation occurs when the subject focuses on an internal representation of the problem situation A and modifies an aspect of it to change it into a representation of situation B, as shown in Fig. 19b. Thus a generative transformation leads initially to the construction of conception B rather than triggering the activity of an established conception B in memory. (In a view which uses the concepts of working memory and long term memory, one would say that a generative transformation can construct conception B from conception A within working memory, whereas an associative leap causes an existing conception B to be moved from long term memory into working memory. However, a commitment concerning whether conceptions in permanent memory are "copied" temporarily into a separate working memory "location" seems non-essential to the present discussion.)

Associative leaps. As shown in Fig. 19c, the basic feature distinguishing an associative leap from a generative transformation is that an established schema B is initially activated in permanent memory as opposed to being constructed via a transformation on A. In an associative leap, schema A may activate schema B in memory either by an established association, by some "family resemblance" process or by the fact that conceptions A and B share some feature(s) in common. In an associative leap, the subject can focus on what is essential and hold it constant, while allowing everything else to change as new schemas are triggered by association (Fig. 19c). In a transformation, on the other hand, the subject can focus on something inessential and change it in the situation, while holding everything else constant (Fig. 19b).

(Figure 19 is highly oversimplified, since it gives the impression that a transformation is a simple replacement, whereas in the case of spatial transformations of entire shapes, (such as "unbending") the process may be a much more distributed and parallel one. Similarly, Figure 19 portrays an association as a single connection, whereas in some cases a much more complicated pattern recognition process may be involved.)

Two memory access processes. The generative transformation and associative leap methods for generating analogies can be interpreted as constituting two different categories of general methods for accessing knowledge structures in permanent memory. Whereas associative leaps always access familiar cases, generative transformations can generate simpler, more analyzable cases. In both cases the ultimate power of the analogy comes from the accessing of knowledge in memory whose relevance ' the problem was not previously comprehended. In the case an associative leap, the knowledge can be triggered directly by a process of association. But in a generative transformation, the analogous case is first constructed via an act of modification, and only then may it trigger the activity of structures in permanent memory (such as the torsion idea) which can analyze it. In both cases, when the problem solver is successful, some relatively creative divergent activity leads to a new insight. But in a leap, the divergent process occurs by association in permanent memory, whereas in a transformation, the divergent process occurs as the subject acts on his temporary representation of the problem and modifies it. Thus the leap and the transformation may represent two fundamentally different mechanisms for accessing relevant knowledge in unfamiliar problem situations.

Is analogy generation a "controlled" process? One can also speculate about whether there is a process which can guide the "direction" in which analogies are generated. We can first ask whether some analogies are less spontaneous than others in the sense that their generation is more controlled. One might expect that some associative leaps would be among the most spontaneously produced analogies since a reasonable model for an associative leap process would involve some sort of spreading activation model for the triggering of an analogous case. Such a process could have a "random" or directionless character. And in fact, instances have been observed in which the generation of a leap seems quite spontaneous as the subject says something like: "Oh, just looking at that reminds me of..."

On the other hand there is evidence that an associative leap can at times be "directional" in the sense of the subject being able to "steer" the analogy generation process toward analogies which are valid and useful. Instances have been observed where the intention to generate a leap in a certain direction seems present before the fact and the subject says something like: "Let's see, what else can I think of that behaves this way?" Such "directional control" might be an important skill which distinguishes those problem solvers who are good at using analogical reasoning from those who are not. For example, S2, says "What else stretches?.." just before he generates several associative leaps such as "rubber bands... molecules...

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polyesters... spiral springs [two-dimensional]". Here the subject appears to be focussing on a particular "important" feature of springs (stretching behavior) while he generates related situations. Determining the nature of the process which identifies such "important" features is an intriguing and unsolved theoretical problem.

One possible answer is that the subject focuses on features involved in causal relations which he believes are responsible for determining values of dependent variables specified in the problem. The strategy for generating a promising analogy would then be to use the inclusion of important causal variables as a constraint while generating an associative leap by searching memory for associated cases. However, this puts the subject in something of a Catch-22 situation. On the one hand, his main goal is to achieve a causal understanding of the original problem powerful enough to allow him to predict an answer. To do this it may be useful to generate an analogous case B. But on the other hand, the very thing that is needed to generate an effective analogous case is a causal understanding of A that can be held constant while associating to new cases. In this situation one needs a causal understanding to help one generate a causal understanding. The resolution of this dilemma may lie in assuming that the subject's initial causal understanding is only a first approximation that is only partially accurate or complete. The analogous case may then provide a model for a more accurate and complete causal understanding. Such a mechanism might allow subjects to bootstrap themselves toward a better understanding of the problem situation.

Generative transformations would appear to involve a different type of intentional control. The subject is assumed to deploy a perceptual motor action (such as "unbending" the coil into a rod) that is not specified by the problem statement. It stands to reason that such an action would usually be an intentional act, although this does not necessarily imply an act directed by a detailed goal, since as we have seen, some transformations can be playful in origin. In contrast to associative leaps, in using a transformation it makes sense to modify features of system A which are <u>not</u> causally important. This leads to the same dilemma that arose in the discussion of associative leaps above, and to the same possible resolution of the dilemma. That is, problem solvers may be able to bootstrap their way toward a more complete understanding by generating an analogy or series of analogies. Thus, while in some cases analogies are generated in a playful and relatively uncontrolled fashion, in other cases there is evidence for some "steering" of the direction of the analogy generation process.

III. THE PRESENCE OF NOVEL CONSTRUCTIONS IN THE SOLUTIONS

One of the results of this study which most interests the author concerns the analogous cases shown in Figures 15 through 18. These are <u>Gedanken experiments</u> or <u>novel constructions</u> in the sense that they are situations which the subject is unlikely to have studied or worked with before. Another unexpected finding from the study is the sheer variety of analogies produced. Two points need to be made here. First, the variety of the analogies in Figs. 2 through 18, and the novelty of the analogies in Figs. 15 through 18 indicates that some significant creative processes were used which produced individualized and novel approaches to the problem. Such creative processes are not often observed in expert solutions to standard low-level text-book physics problems. But when given a problem like the spring problem, where the expert problem solver has no established, ready-made procedures to apply, creative processes do come into play, and a wide variety of solution "species" evolve.

Secondly, the significance of the novelty of the cases in Figs. 15 - 18 is that it argues that they were at least in part invented by the subject rather than recalled directly from memory. A very common view of analogous cases is that they are conceptions "sitting in memory" which are at some point activated or recalled as being possibly related to a current problem. The data reported here indicate that this may be true of some analogies but not all. In some instances subjects can actually construct an analogous case. Such cases suggest that an analogy is not always simply "recognized" between two existing conceptions. Thus, the analogies observed do not always consist of familiar cases recalled from permanent memory. The analogies can also consist of newly-constructed invented cases.

<u>Limitations of this analysis</u>. The reader will recall that four processes were identified as necessary to making an inference by analogy: (a) generating the analogy; (b) confirming the analogy relation; (c) comprehending the analogous case; and (d) transferring findings from the analogous case back

to the original case. The present paper has concentrated only on the first process, analogy generation. There are many other creative reasoning patterns in the protocols of expert scientists that have not yet been adequately described here, including: the recursive use of analogies to evaluate and confirm previous analogies, leading to the formation of chains of analogies; the use of symmetry arguments; and the special forms of reasoning involved in extreme case arguments, among others. (See Clement, 1982a, 1982b, for a preliminary description.) We have only scratched the surface of the phenomena to be studied in this area.

EDUCATIONAL IMPLICATIONS

Research on analogical reasoning may have important educational implications for two reasons. First, scientific models play a central role in learning science. Many models in science can be viewed as highly developed analogies [6, 7, 9]. This suggests that analogical reasoning should play a significant role in the process of understanding scientific models [19]. Secondly, analogy may play a central role in knowledge transfer during problem solving. One of the most important remaining questions in problem solving research today is: "How does one apply existing knowledge to an unfamiliar problem situation?" Teachers often complain that students have difficulty solving new problems that vary only slightly from those they have solved already. A plausible way in which one could successfully solve unfamiliar problems would be to form an analogy to an appropriate key example that one has already learned about. Thus the ability to make appropriate analogies appears to be of central importance in the process of knowledge transfer.

There is some reason to believe that one can train students to use analogical reasoning. Previous studies of freshman engineering students have shown that analogies are a natural form of reasoning for many subjects [4, 5]. Often these analogies are insufficiently developed or invalid, but nevertheless the motivation and ability to generate analogies appears to be present. Many expert solutions by analogy are not "instant solutions", but involve a more extended process of conjecture and testing [2, 3]. These findings give us reason to believe that some of these processes are learnable, rather than being exclusively a product of "genius", and that developing students' abilities to use generative transformations, associative leaps, and extreme cases may be possible and desirable.

CONCLUSION

Scientifically trained individuals can and do use analogical reasoning in solving problems. Furthermore, they exhibit more than one method for generating analogies. Apparently analogies can be generated by an associative process which triggers the new involvement of an old but separate idea, or they can be generated by a transformation process which modifies the current problem situation. The novelty of some of the cases generated via transformations indicates that they are constructed rather than being recalled from memory. Both methods for generating analogies are creative acts, and are important potential sources of creative power in scientific thinking. These findings have significant educational implications for the learning of scientific models and the transfer of knowledge to new problem situations.

NOTES

1. It should be emphasized that by generation via a principle we mean a formal principle or equation. In future studies one may have to posit a continuum involving the extent to which the original concrete problem representation is complimented by other more abstract representations such as (1) general categories of situations, (2) physical intuitions, (3) imageable mathematical models (such as graphs), (4) formal principles expressed verbally, such as Newton's Laws, or (5) equations. For the purpose of this study, the analogy was considered to be generated via a formal principle if there was evidence that the analogy stemmed from one of the latter three representation types.

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SPRING PROBLEM

A WEIGHT IS HUNG ON A SPRING. THE ORIGINAL SPRING IS REPLACED WITH A SPRING

- --MADE OF THE SAME KIND OF WIRE,
- --WITH THE SAME NUMBER OF COILS,
- --BUT WITH COILS THAT ARE TWICE AS WIDE IN DIAMETER.

WILL THE SPRING STRETCH FROM ITS NATURAL LENGTH, MORE, LESS, OR THE SAME AMOUNT UNDER THE SAME WEIGHT? (ASSUME THE MASS OF THE SPRING IS NEGLIGIBLE COMPARED TO THE MASS OF THE WEIGHT.) WHY DO YOU THINK SO?

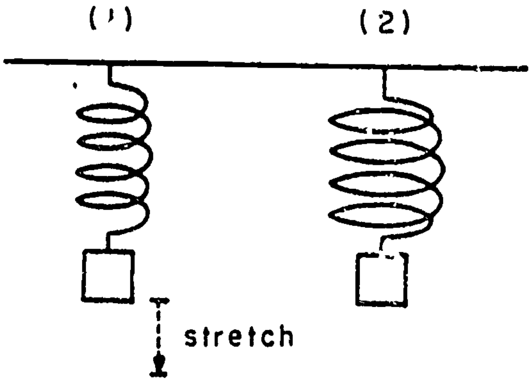


FIGURE 1

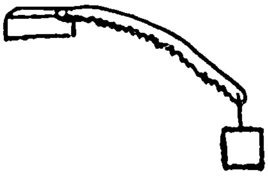


Fig. 2

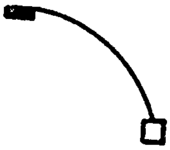
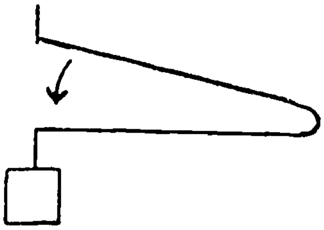


Fig. 3



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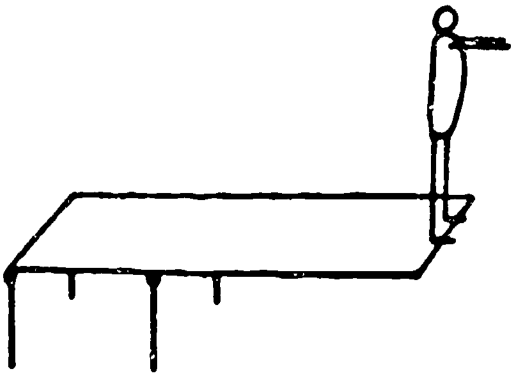

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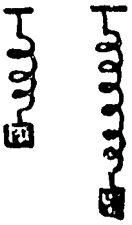

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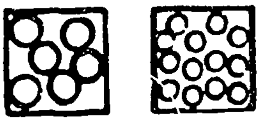

Fig. 7

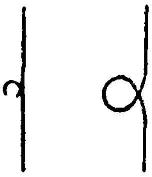

Fig. 8

Some Analogies Generated by Subjects Solving the Spring Problem

Fig. 2	Longer Sawblade Bends More
Fig. 3	Longer Rod Bends More
Fig. 4	Longer Hairpin Bends More
Fig. 5	Longer Diving Board Bends More
Fig. 6	Longer Spring Stretches More
Fig. 7	Foam Rubber with Larger Air Holes Compresses More
Fig. 8	Larger Kinks in a Wire Easier to Remove

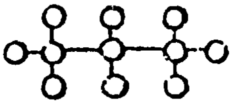

Fig. 9

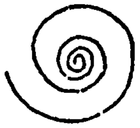

Fig. 10

Fig. 11

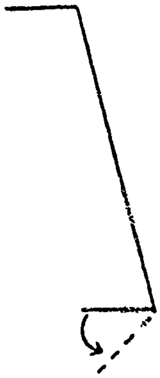

Fig. 12

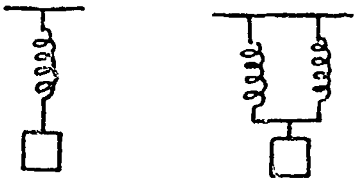

Fig. 13

Fig. 14

Some Analogies Generated by Subjects Solving the Spring Problem

Fig. 9	Polyesters
Fig. 10	Spiral Spring in Two Dimensions
Fig. 11	Car Spring
Fig. 12	Longer Rod Twists More Under Same Torque
Fig. 13	Parallel Springs Stretch Less
Fig. 14	Mountain Roads. Car climbs farther per turn on road on wider mountain, given the same incline angle. So assuming same weight produces same helix incline, wide spring would stretch more.

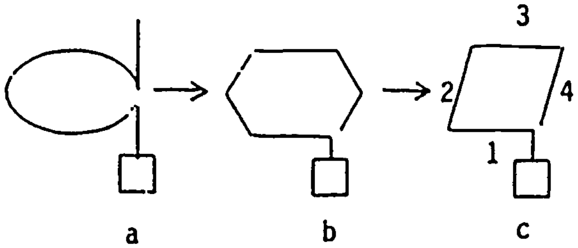

Fig. 15

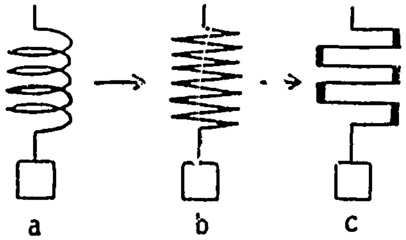

Fig. 16

Fig. 17

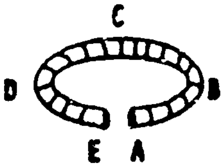

Fig. 18

Novel Analogous Cases Constructed by Subjects

Fig. 15 Hexagonal and Square Coils (Leading to Torsion Insight)

Fig. 16 Two Dimensional Zig-Zag Spring and Modified Zig-Zag with Stiff Joints

Fig. 17 Pitting the Wide Spring Against the Narrow Spring

Fig. 18 Torsionless Coil with Bearings Between Elements

ANALOGY GENERATION VIA A PRINCIPLE

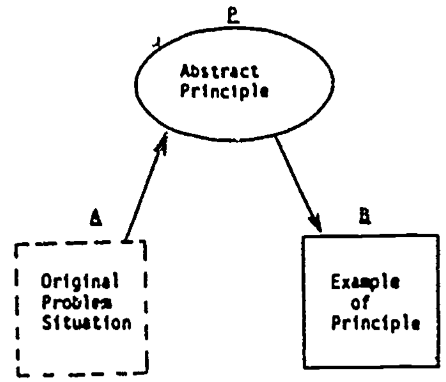

Fig. 19A

ANALOGY GENERATED VIA A TRANSFORMATION

TRANSFORMATION MODIFIES CONCEPTION

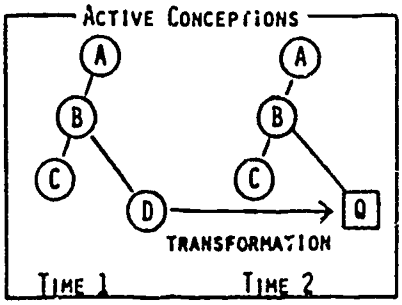

Fig. 19B

KEY:

○ SCHEMA ELEMENTS WHICH ARE THE SAME

□ SCHEMA ELEMENTS WHICH ARE DIFFERENT

GENERATION VIA AN ASSOCIATIVE LEAP

NEW SCHEMA ACTIVATED ASSOCIATIVELY IN MEMORY

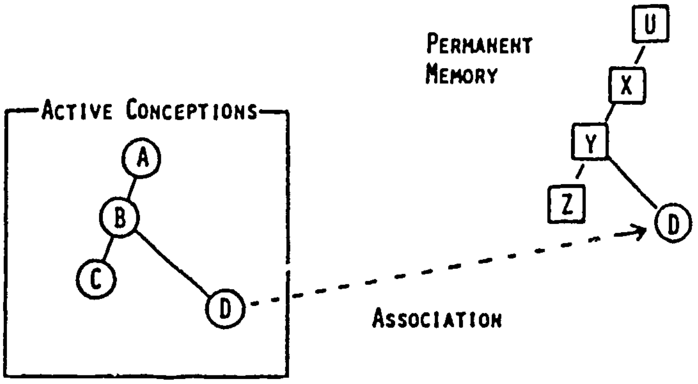

Fig. 19C

SPONTANEOUS ANALOGIES GENERATED FOR THE SPRING PROBLEM

N = 10

Total Number of Analogies Generated	Number of Salient Analogies Generated
40	32

Number of Subjects Generating at Least One Analogy	Number of Subjects Generating a Salient Analogy
8	7

Table 1

ANALOGY GENERATION METHODS FOR THE SPRING PROBLEM (N = 10)

Total Number of Salient Analogies Generated
32

Generation Mechanisms

Transformations	Associative Leaps	Via a Principle	Method Unclear
19	8	1	4

Table 2